King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 254: Discrete Structures II
Spring Semester 2020-2021 (202)
Final Exam, Tuesday April 27th, 2021.

Time: 120 Minutes

IMPORTANT:

1. **Include your Name and ID# on Every Answer Page.**
2. **This exam consists of 13 questions. You have to answer all 13 questions.**
3. **The exam is closed book and closed notes. NO CALCULATORS or any helping aides are allowed.**

1) (6 points) Consider $A = \{1, 2, 3, 4\}$. Find the smallest relation containing the relation $R = \{(1,2), (1,4), (3,3), (4,1)\}$ that is both reflexive and symmetric.

$\hat{R}$

$\hat{R} = R \cup \{(1,1), (2,2), (4,4), (2,1)\}$

2 4

-1 for each missing element

* R is not included -2
* one element is missing -1 per each
* one wrong pair added -1 each

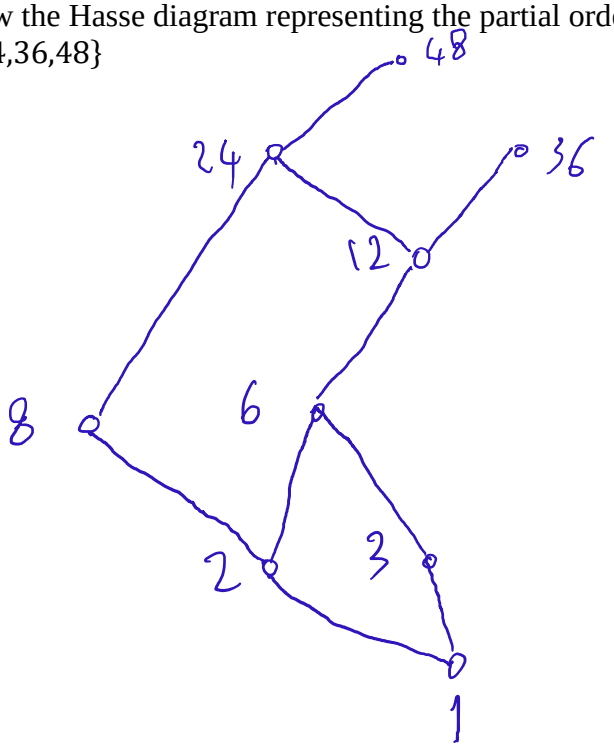
2) (9 points) Show that the relation R consisting of all pairs (x, y) such that x and y are bit strings of length three or more that agree except perhaps in their first three bits is an equivalence relation on the set of all bit strings of length three or more.

1. Reflexive
$(x,x) \in R$ $\forall x$ where x is a bitstring of length 3 or more since x agrees with itself in all bits

2. Symmetric
$(x,y) \in R \rightarrow$ x & y agree except on the first 3 bits. If that is the case, then (x,y) also agree except in the first 3 bits (possibly).

3. Transitive:
$(x,y) \in R$ & $(y,z) \in R$.
x, y agree except on first 3 bits
y, z " " " " " "
∴ x, z " " " " " "
∴ $(x,z) \in R$.
∴ R is an equivalence relation.

3) (10 points) Draw the Hasse diagram representing the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on {1,2,3,6,8,12,24,36,48}



-1 for each missing arc

-1 for each wrong arc.

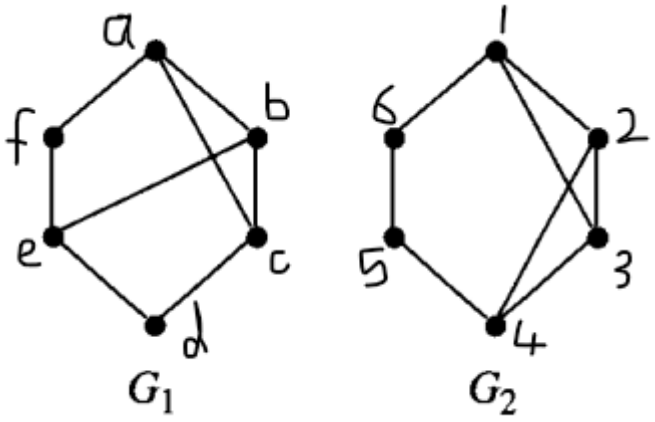
4) (5 points) Determine whether the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1,2,3,6,8,12,24,36,48\}$ is a lattice or not. Justify your answer.

Not a lattice

Since (36, 48) does not have a least upper bound

or (8, 36)

5) (6 points) Determine whether the following graphs are isomorphic or not. Prove your answer.

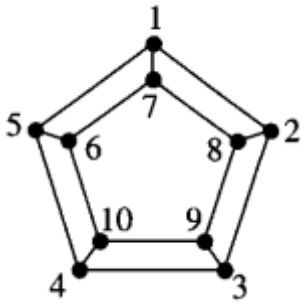


2 — Not isomorphic

G_1 has 1 cycle of length 3 (abca)

4 — G_2 has 2 cycles of length 3 (1231 & 2342)

6) Answer the following questions based on the following graph:



a) (6 points) Find a Hamiltonian cycle in the above graph.

1 5 4 3 2 8 9 10 6 7 1

<6, 0 7

b) (6 points) Does the above graph have an Euler circuit? Justify your answer.

No. There are more than 1 vertex with odd degree

(3) (3)

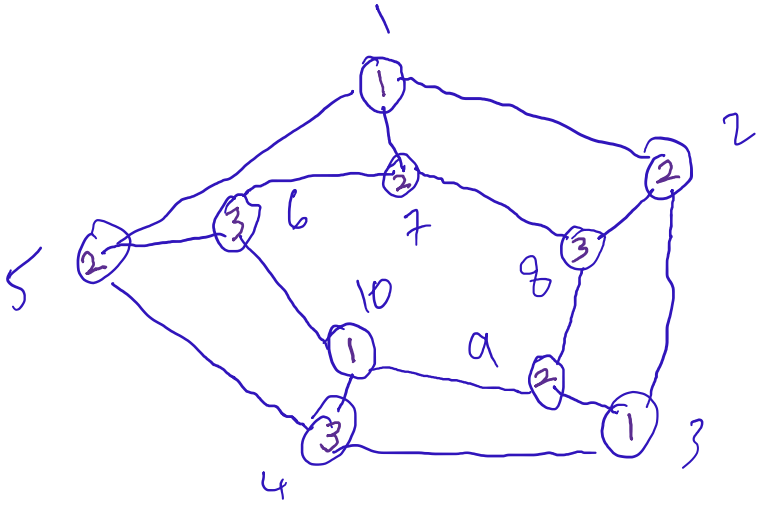
c) (6 points) Show that the graph satisfies Euler's formula relating the number of edges, vertices and regions for planar graphs.

edges = 15 = e
vertices = 10 = v
regions = 7 = r

(1) (1) (1) (3)

r = e − v + 2
7 = 15 − 10 + 2 ✓

d) (6 points) Prove that the chromatic number of the above graph is 3.



<6, 6 7

7) (5 points) Compute the value of the following prefix expression, showing all the intermediate steps. A final answer without steps is worth zero points.

+ − 10 8 ∗ 3 − 7 2

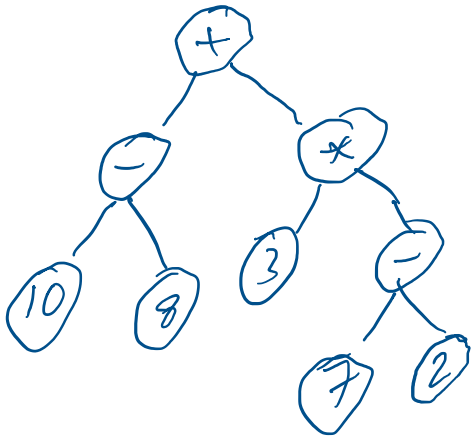
− 7 2 = 5

+ − 10 8 ∗ 3 5

+ − 10 8 15

+ 2 15

17



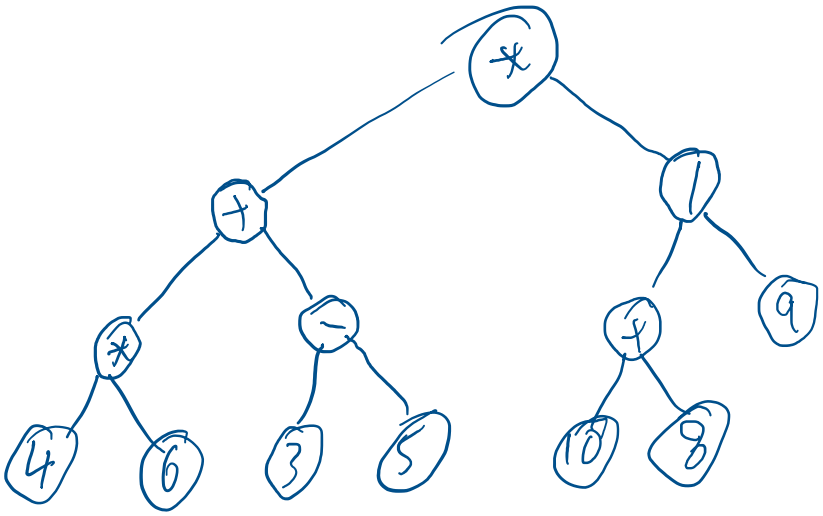
Mistake in 1 operation −1

" 2 operations −3

" −5

8) (5 points) Draw the ordered rooted tree corresponding to the postfix expression

$4\ 6 * 3\ 5 - + 10\ 8 +\ 9\ / *$



9) (5 points) Assume that $\Sigma = \{a, b\}$. For each of the following regular expressions, find a string $x \in \Sigma^*$ that is not recognized by the given regular expression. If all strings are recognized, write "None."

a) $(ba + b + aa)^*$ {a, ab, aaa, ... }

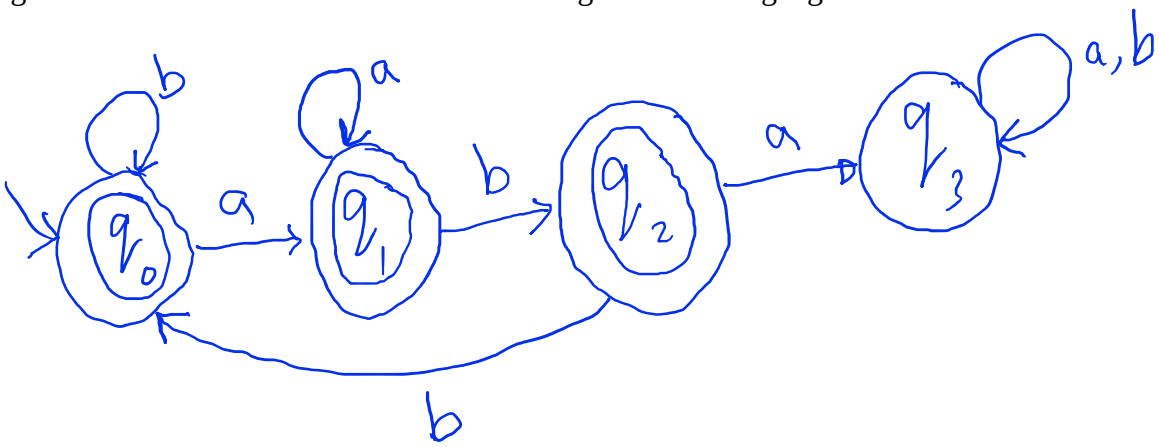
b) $(b^*a^*)^*$ None

10) (5 points) Let $\Sigma = \{a, b\}$. Consider the language consisting of the set of strings that contain an even number of a's. Give a regular expression for this language.

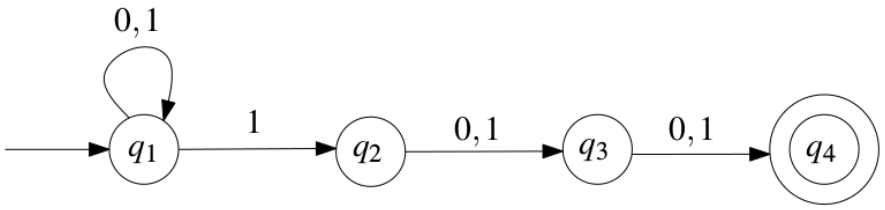
(b* + (b*ab*ab*)*)

OR (ab*a + b)* OR any correct answer.

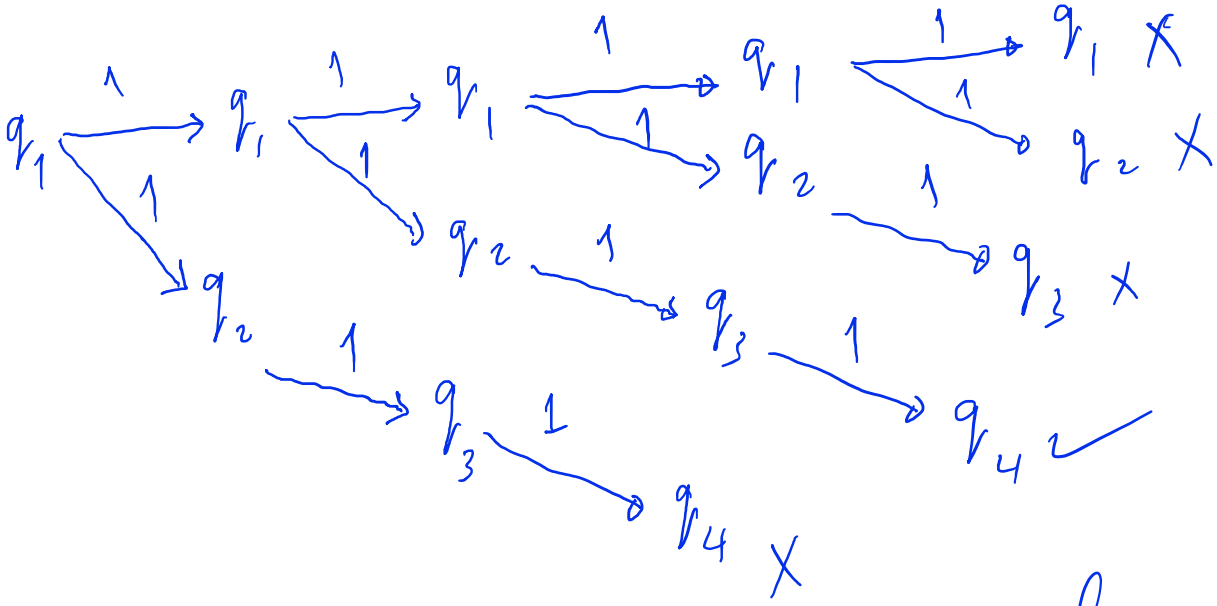
11) (5 points) Let $\Sigma = \{a, b\}$. Consider the language consisting of the set of strings that does not contain *aba*. Design a deterministic finite automaton that recognizes this language.



12) (5 points) Consider the following nondeterministic finite state machine



Determine whether the string 1111 belongs to the language recognized by the above nondeterministic finite automaton or not, by showing the propagation of states for the string.



0 0 1111 belongs to the language.

13) (10 points) Construct a non-deterministic finite automaton for the regular expression $(ba + c)^* b$